REPORT

(to D-Flex Ltd, NWDA Innovation Vouchers Award, 28 April 2009)

Finite Element Modelling of Rigidity of Fishing Rods with Various Cross-sections

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Summary

D-Flex Ltd has developed and patented a novel design of fishing rod that is based upon adopting a non-circular cross sectional shape which is a Curve of Constant Width, in particular an extended Reuleaux Triangle, to beneficially enhance flexural rigidity. In this report a basic evaluation of the bending of tubes was undertaken to simulate how fishing rods with various cross-sectional shapes might behave. Finite element models were developed and the rigidity of representative tubes was determined in terms of deflection under a static load. Dimensional parameters were chosen to ensure a matched volume of material per unit length. The shapes evaluated were circular, equilateral triangle, Reuleaux Triangle and extended Reuleaux Triangle. An elastic constitutive model was employed and nominal material properties were used. Deformed shapes of tubes are presented and discussed. It was found that all the non-circular shaped tubes demonstrated increased rigidity (reduced deflection) when compared to a circular tube and that the extended Reuleaux Triangle was the most efficient shape of those investigated. In addition, different elastic modulii, representative of the variable compression/tension properties of the carbon fibre composite materials typically used in fishing rods, were incorporated into the tension and compression zones of an extended Reuleaux tube and loadings through its apex and side were modelled. This revealed different rigidities in relation to these two loading conditions and thus that there is a differential in the flexural properties of such a tube depending upon the direction of loading. Overall, it was found that an extended Reuleaux Triangle tube will demonstrate a reduced deflection of approximately 25% less than an otherwise equivalent circular section tube.

Introduction

For a fishing rod, it is desirable to maximise its rigidity for a given cross-sectional area (volume of material used) and one way to do this is by varying the cross-sectional shape. The advantage for doing that is either to enhance rigidity of a fishing rod or to reduce the material used but still maintain a necessary rigidity. If such shape can be identified it will be of great interest to fishing rod manufacturers and other manufacturers who are interested in shape optimisation.

Initial physical testing of prototypes (application of a dead load) with two cross-sectional shapes has shown that the extended Reuleaux Triangle (a Reuleaux Triangle with each apex rounded to reduce stress concentration, with a radii ratio of 5:1) is a superior shape which enhances rigidity over 20% in comparison to a circular shape. However, such initial tests only provided approximated estimations and there was no rigorous scientific backup. In addition shapes other than circular and extended Reuleaux shapes need to be evaluated also. Therefore it was proposed to develop computer models using the finite element analysis technique to systematically investigate rigidities of fishing rods with various cross-sectional shapes covering circular, equilateral triangle, Reuleaux and extended Reuleaux geometries.

For the purposes of this study, the wall thickness was fixed as 1 mm and the overall length as 1000 mm. Only linear elasticity was employed, using a representative modulus of elasticity of 1500 GPa and Poisson's ratio as 0.15. The base cross-sectional area was taken from a circular shape with external diameter as 10 mm, i.e.

$$A_{circular} = \pi(5^2 - 4^2) = 28.2743mm^2$$

As the length is fixed as 1000 mm, the initial volume will be the same to all other shapes investigated as long as the cross-sectional area is initially same. Figure 1 shows all shapes to be investigated.

For the equilateral triangle shown in Figure 1b, in order to have the same cross-sectional area as that of the circular one there is

$$3\left[(a-3.4642)\times 1 + 2\times\frac{1}{2}\times 1.7321\times 1\right] = 28.2743 \tag{1}$$

which gives *a = 11.1569 mm*.

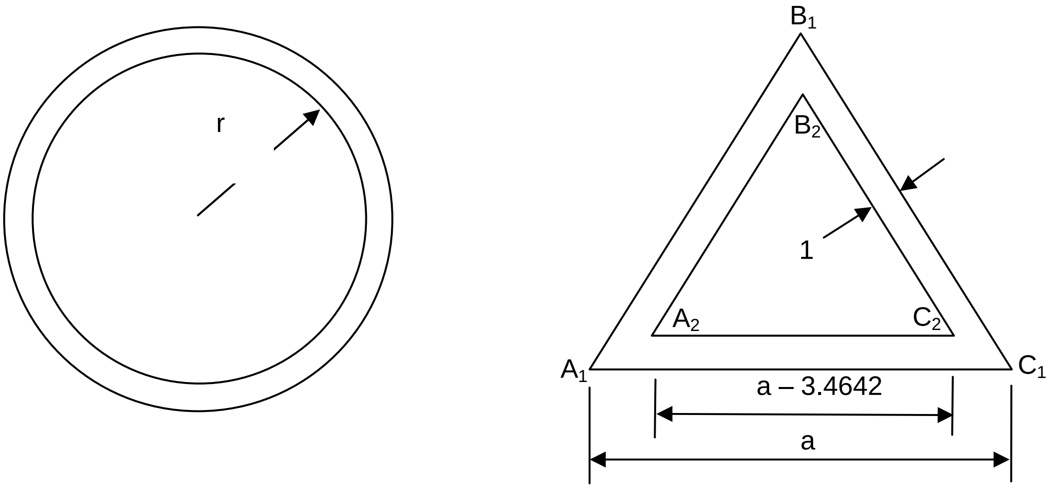
For the Reuleaux shape shown in Figure 1c, by using the Sine rule in the triangle $A_1A_2C_1$ and letting the cross-sectional area equal to that of the circular one (28.2743 mm^2), there are

$$\frac{r_1}{\sin 30^o} = \frac{r_1 + 1}{\sin(150^o - \theta)} \tag{2}$$

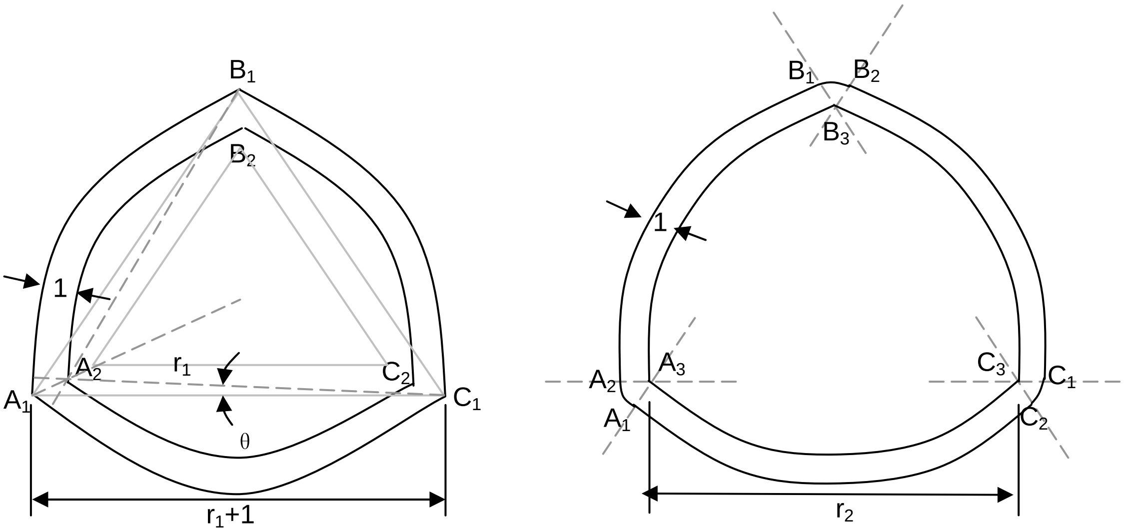
$$\frac{\pi}{120}((60^o - 2\theta)[(r_1+1)^2 - r_1^2] + (60^o + 2\theta)\times 1^2) = 28.2743 \tag{3}$$

Solving Eqs. (2) and (3) using the Matlab, there are

$$r_1 = 9.1244mm, \theta = 3.6968^o \tag{4}$$



(a) circular, (b) equilateral triangle



(c) Reuleaux, (d) extended Reuleaux

Figure 1. Cross-sectional shapes investigated.

For the extended Reuleaux shape shown in Figure 1d, similarly there is

$$\frac{\pi}{2}[((r_2+1)^2 - r_2^2) + 1^2] = 28.2743 \tag{5}$$

which gives

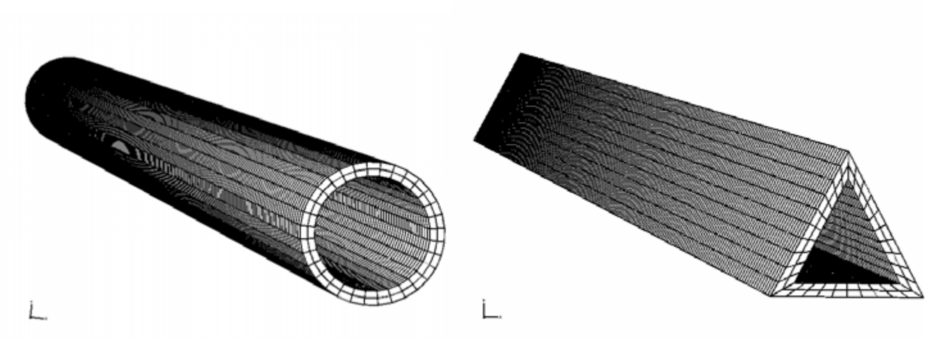
$$r_2 = \frac{A_{circular}}{\pi} - 1 = 8.0\,mm \tag{6}$$

Development of finite element models

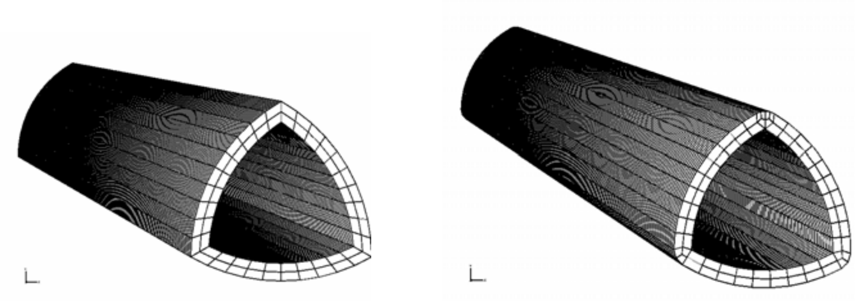
Using geometries developed the above, coordinates of key points for the shapes other than circular one are shown in Table 1. Mesh generations were then carried out to cover all four cross-sectional shapes, which are shown in Figure 2. Eight-node iso-parametric solid elements were used to descretize four different rods with typical element size as 0.5mmx1.8mmx5mm. Two elements were created through the wall thickness. A concentrated load of 40 N was applied to the free end of the rod, and fixed boundary conditions were applied to the other end of the rod in 50mm length. The load increment was controlled as 5% of the total load.

Table 1. Coordinates of the key geometric points in mm.

Shape		A$_1$	A$_2$	A$_3$	B$_1$	B$_2$	B$_3$	C$_1$	C$_2$	C$_3$
triangle	x	-5.5784	-3.8464		0.0000	0.0000		5.5784	-3.8464	
	y	-4.2207	-3.2207		6.4414	4.4414		4.2207	-3.2207	
Reuleaux	x	-5.0622	-4.0432		0.0000	0.0000		5.0622	-4.0432	
	y	-2.9027	-2.3344		5.8453	4.6687		2.9027	-2.3344	
ex-Reuleaux	x	-4.5000	-5.0000	-4.0000	-0.5000	0.5000	0.0000	5.0000	4.5000	4.0000
	y	-3.1754	-2.3094	-2.3094	5.4848	5.4848	4.6188	-2.3094	-3.1754	-2.3094



(a) circular, (b) equilateral triangle,

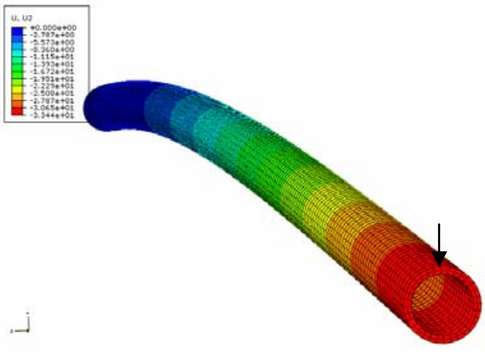


(c) Reuleaux, (d) extended Reuleaux,

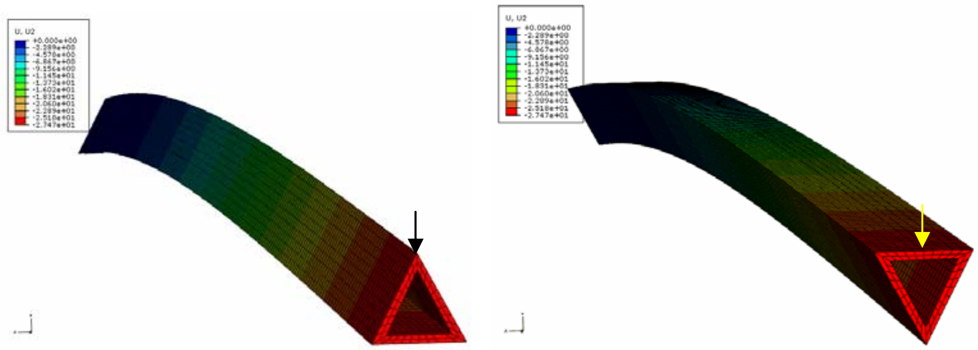
Figure 2. Mesh generations.

Results and discussion

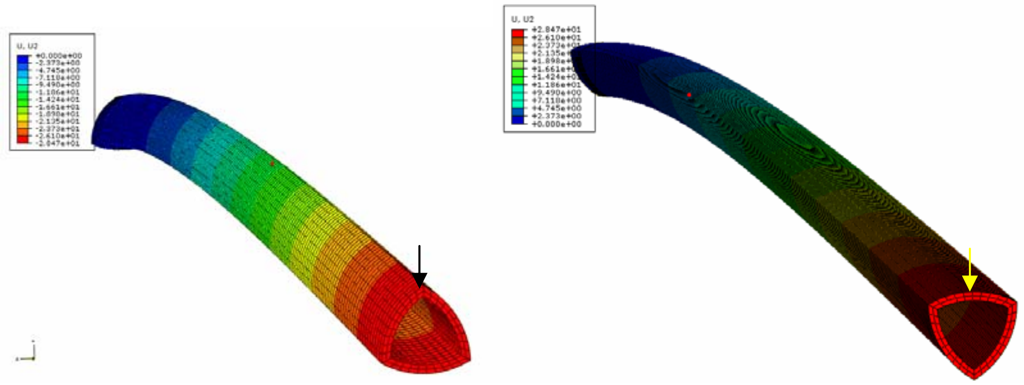
All models developed were submitted to a finite element package. Figure 3 shows



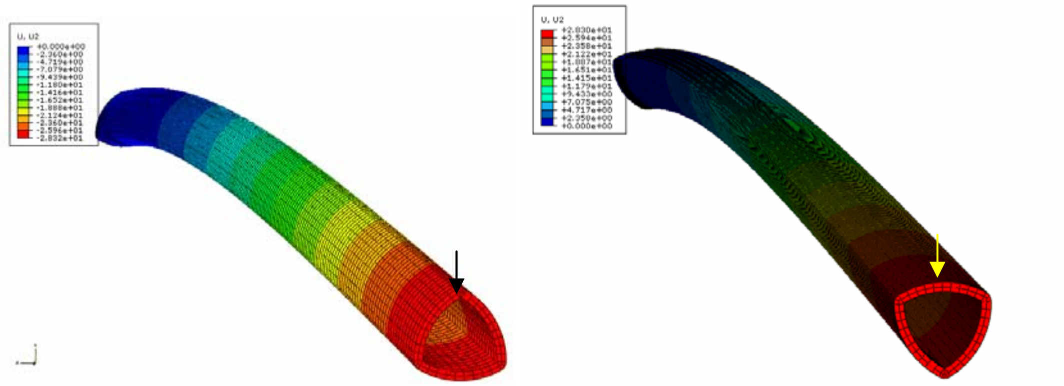
(a). circular,



(b). triangle loading on apex, (c). triangle loading on side,



(d). Reuleaux-apex, (e). Reuleaux-side,



(f). extended Reuleaux-apex, (g). extended Reuleaux-side,

Figure 3. Deformed shapes of all cross-sectional shapes investigated.

deformed shapes of five rods investigated, which covers circular, equilateral triangular (loading on the apex and the side), Reuleaux, and extended Reuleaux shapes. The corresponding deflections at the free end are displayed in Table 2.

Table 2. FE results.

	circular (base)	tri-apex	tri-side	Reuleaux-apex	Reuleaux-side	ex-Reuleaux apex	ex-Reuleaux side
deflection(mm)	33.44	27.47	27.47	28.47	28.47	28.37	28.37
variation (%)	0	-17.85	-17.85	-14.86	-14.86	-15.16	-15.16
max stress (Mpa)	778.1	832.7	832.7	740.0	747.7	738.0	759.5
variation (%)	0	+7.0%	+7.0%	-5.0%	-3.9%	-5.2%	-2.4%

If take the circular cross-section as a base shape, it can be seen that the deflections for other 4 shapes are significantly reduced, i.e. from 14.86% reduction for the Reuleaux to 17.85% reduction for the equilateral triangle. It is understandable why the deflections are same for both rods with equilateral triangle subjected to loads through the apex and the side respectively, as the area moments for both situations are exactly same. This also applies to Reuleaux and extended Reuleaux shapes. As long as the load is acting through an apex or a side that are located in a symmetrical line of a cross-section, the corresponding deflections are expected to be same due to the same area moment. However, the maximum stress level may be varied slightly. Although the deflection from the triangular shape is the lowest, such shape is not suitable for a fishing rod as apex does not give comfortable handling to the user. In addition, its maximum Mises stress level (a parameter which indicates the cumulative of stresses experienced) is higher than those of other shapes, i.e. 7% higher in comparison with the circular one and over 12% in comparison with the Reuleaux and the extended Reuleaux shapes.

Practically, modulus of elasticity for carbon fibre laminates is different in tension and compression, which will give different rigidities when an extended Reuleaux tubular rod is subjected to loading through its apex and side. Taking the compressive modulus as the nominal one used and assuming the tensile modulus 30% higher than the compressive one, deflections corresponding to loads acting through the apex and the side are 24.93 mm and 25.38 mm, respectively. Figure 4 shows the tension and compression areas where different modulii were used. The areas of the tension zone and the compression zone are 48% and 52% of the total cross-sectional area respectively for loading through the apex, and vice versa for loading through the side. This was verified by handling a fishing rod in the orientations corresponding to the loading cases displayed in Figure 4. However, in the numerical modelling only a 1000 mm long tubular rod with constant cross-section was calculated. This inevitably provides proportionally less difference in deflections (rigidities) between two loading cases, in comparison with handling a full-length fishing rod with a tapered cross-section. It is expected that such difference will be magnified when modelling a long tubular rod with variable material properties and a tapered section.

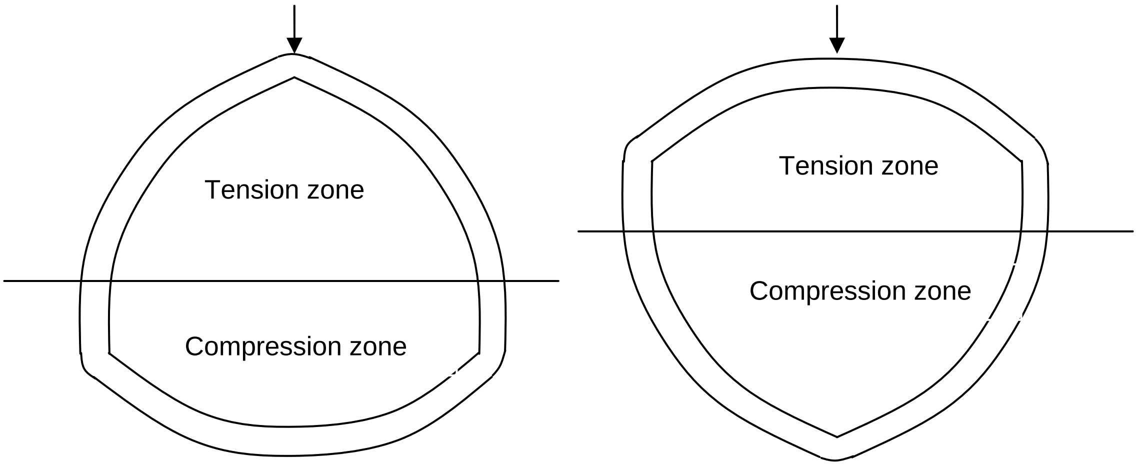


Figure 4. Tension and compression zones for applying the related elastic modulii.

It is worth pointing out that the extended Reuleaux shape has a great potential to enhance rigidity by either slightly increasing its cross-sectional area (so the overall cross-dimension) or maintaining its cross-sectional area but reducing its wall thickness slightly (so to increasing the overall cross-dimension), yet still maintaining a relatively low stress level. The purpose for the above approaches is to increase the overall cross-dimension so that its area moment can be increased. As an example, a trial run of the extended Reuleaux was carried out by increasing its cross-sectional area from 28.27 mm^2 to 30.33 mm^2 and its r_2 (see Figure 1d) from 8.00 mm to 8.66 mm. The corresponding deflection was reduced from 28.37 mm (see Table 2) to 23.08 mm. This is an 18.65% reduction in deflection with only 7.29% increase in the cross-sectional area. If compare it with the circular shape, the reduction in deflection is 30.98%. Yet, the Mises stress level is also decreased to 641.3 MPa, which is a 17.58% reduction.

By systematically undertaking parametric studies, optimisation of the extended Reuleaux shape can be obtained to achieve either material savings by reducing the wall thickness but maintaining the similar rigidity to the circular shape or rigidity enhancement by reducing the maximum deflection but using the same cross-sectional area to the circular one. However, due to the time limitation in the current project, such optimisation could not be carried out.

Conclusions

Finite element models have been developed to investigate the maximum deflections for various tubular rods with cross-sections covering circular, equilateral triangular, Reuleaux and extended-Reuleaux shapes. It has been found that the extended Reuleaux shape is a most efficient shape in terms of rigidity and stress level. Although the equilateral triangular shape gives slightly lower deflection, it is not user friendly and would not be practical to manufacture. The extended Reuleaux shape, however, is easy to incorporate within established manufacturing processes, which are based upon rolling sheet material around a forming mandrel, and provides huge potential for material savings and rigidity enhancement in fishing rods. Also, exploitation of different material properties in tension and compression plays an important role to give extended Reuleaux rod users different rigidities when using the rod in different orientations.

Acknowledgement

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